Quiz #1A, MTH 1410, Spring 2013

3:04
3:07
3 min.

Name: Key

1. (3 points) For what value of c is the function continuous at $x = 4$? **Explain your reasoning.** To receive full credit, you must use correct notation and the definition of continuity.

$$f(x) = \begin{cases} \dfrac{(x-2)^2 - 4}{x-4} & \text{if } x < 4 \\ 5c & \text{if } x \geq 4 \end{cases}$$

Need $\lim_{x \to 4} f(x) = f(4)$

$$f(4) = 5c = \lim_{x \to 4^+} f(x)$$

$$\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \frac{(x-2)^2 - 4}{x-4}$$

$$= \lim_{x \to 4^-} \frac{x^2 - 4x + 4 - 4}{x-4}$$

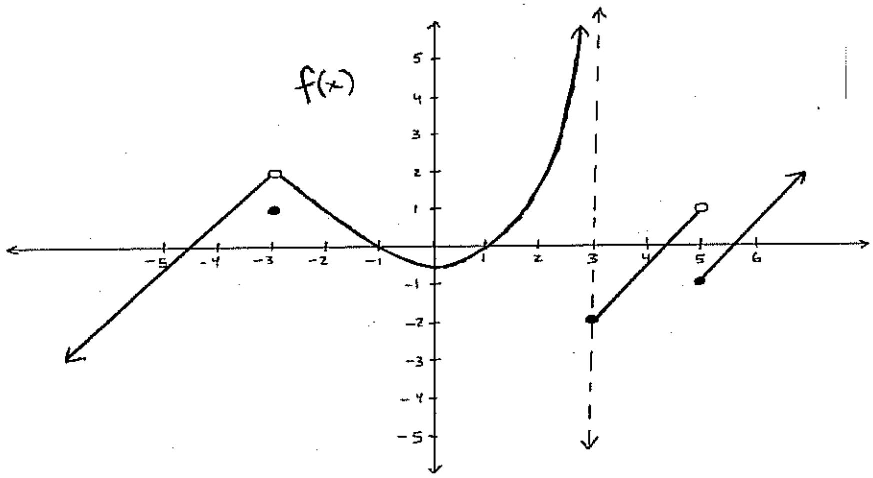
$$= \lim_{x \to 4^-} \frac{x\cancel{(x-4)}}{\cancel{x-4}}$$

$$= \boxed{4}$$

$\Rightarrow$ Need $4 = 5c$

$$\boxed{\frac{4}{5} = c}$$

2. (3 points) For the given graph, calculate the limit or state that it does not exist. If it does not exist, (briefly) explain why.



(i) $\lim_{x \to (-3)} f(x) =$ 2

(ii) $f(-3) =$ 1

(iii) $\lim_{x \to 3^-} f(x) =$ dne

(∞)

graph keeps going up.

(iv) $\lim_{x \to 5^+} f(x) =$ -1

3. (4 points) Calculate the limit, if it exists. If it does not exist, explain why. Hint: Rationalize (that is, multiply by the conjugate) and simplify. Make sure you use correct notation!

$$\lim_{x \to 6} \frac{\sqrt{x-5}-1}{x-6} \cdot \frac{\sqrt{x-5}+1}{\sqrt{x-5}+1}$$

$$= \lim_{x \to 6} \frac{(x-5)-1}{(x-6)(\sqrt{x-5}+1)}$$

$$= \lim_{x \to 6} \frac{\cancel{x-6} \quad 1}{\cancel{(x-6)}(\sqrt{x-5}+1)}$$

$$= \frac{1}{\sqrt{6-5}+1} = \boxed{\frac{1}{2}}$$

Quiz #1B, MTH 1410, Spring 2013

Name: Key

1. (4 points) Calculate the limit, if it exists. If it does not exist, explain why. Hint: Rationalize (that is, multiply by the conjugate) and simplify. Make sure you use correct notation!

$$\lim_{x \to 11} \frac{\sqrt{x-2}-3}{x-11}$$

$$= \lim_{x \to 11} \frac{(\sqrt{x-2}-3)}{x-11} \cdot \frac{(\sqrt{x-2}+3)}{(\sqrt{x-2}+3)}$$

$$= \lim_{x \to 11} \frac{x-2-9}{(x-11)(\sqrt{x-2}+3)}$$

$$= \lim_{x \to 11} \frac{\cancel{x-11}\ 1}{\cancel{(x-11)}(\sqrt{x-2}+3)}$$

$$= \frac{1}{\sqrt{11-2}+3}$$

$$= \boxed{\frac{1}{6}}$$

2. (3 points) For what value of c is the function continuous at $x = \overset{2}{\cancel{A}}$? **Explain your reasoning.** To receive full credit, you must use correct notation and the definition of continuity.

Type

$$f(x) = \begin{cases} \dfrac{(x-1)^2 - 1}{x-2} & \text{if } x < 2 \\ 3c & \text{if } x \geq 2 \end{cases}$$

Need $\lim_{x \to 2} f(x) = f(2)$ $\qquad f(2) = 3c$

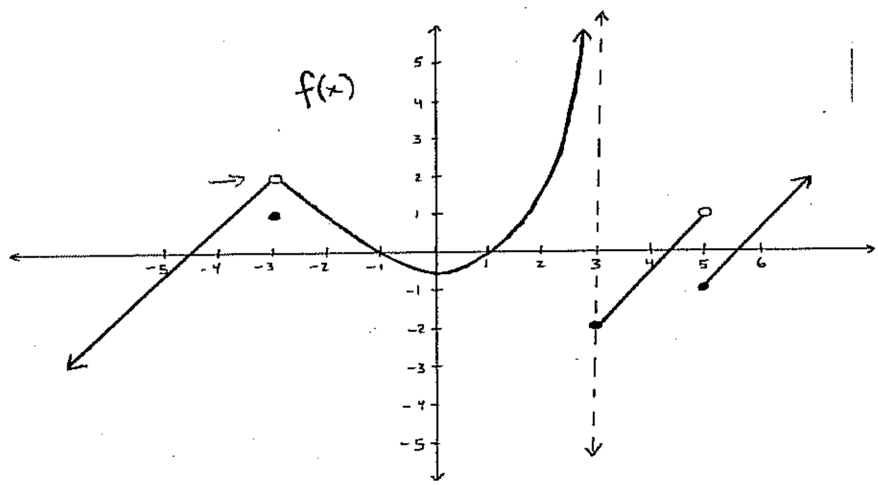
$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \frac{(x-1)^2 - 1}{x-2}$$

$$= \lim_{x \to 2^-} \frac{x^2 - 2x + 1 - 1}{x-2}$$

$$= \lim_{x \to 2^-} \frac{x\cancel{(x-2)}}{1\cancel{(x-2)}} = 2$$

$\Rightarrow$ Need $2 = 3c$, so $\boxed{c = \frac{2}{3}}$

3. (3 points) For the given graph, calculate the limit or state that it does not exist. If it does not exist, (briefly) explain why.



(i) $\lim_{x \to (-3)^-} f(x) = 2$

(ii) $\lim_{x \to (3)^+} f(x) = -2$

(iii) $\lim_{x \to 5} f(x) =$ DNE b/c

$\lim_{x \to 5^-} f(x) = 1$, $\lim_{x \to 5^+} f(x) = -1$

and $1 \neq -1$

(iv) $f(5) = -1$